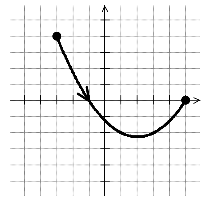
How to change the direction or speed of traversing a parametric curve

Supposed you already have parametric equations for a curve with a finite domain for the parameter,
but you want to change the direction and/or speed of travelling along the curve.
This can be accomplished by "changing the timeline".

For example,

the parametric equations $\begin{aligned} x &= 2t-1 \\ y &= t^2-3t \end{aligned}$ for $t \in [-1, 3]$ correspond to the curve on the right.



At time $t=-1$, the curve starts at $(x, y) = (-3, 4)$ (the leftmost point) and
at time $t=3$, the curve ends at $(x, y) = (5, 0)$ (the rightmost point).
In between, the curve was traversed from left to right.

Suppose you want to traverse the curve from right to left instead,
and instead of taking $3-(-1)=4$ units of time to do so,
you want to take only 2 units of time.

Create a new time variable T and determine how the values of T correspond to the values of the original time variable t.

In this case, let's say you want $T \in [0, 2]$ (so T spans $2-0=2$ units of time).
At $T=0$, you want to start the curve at $(x, y) = (5, 0)$, which corresponds to $t=3$.
At $T=2$, you want to end the curve at $(x, y) = (-3, 4)$, which corresponds to $t=-1$.

So, $t=3$ when $T=0$, and $t=-1$ when $T=2$.

Now find a linear function for the original time variable t in terms of the new time variable T.

$$t = mT + b$$
$$m = \frac{\Delta t}{\Delta T} = \frac{3-(-1)}{0-2} = -2$$
$3 = -2(0) + b$, so $b = 3$

So, $t = -2T + 3$

Substitute this expression for the original time variable into the original parametric equations
to get new equations in terms of the new time variable.

$$\begin{aligned} x &= 2t-1 \\ y &= t^2-3t \end{aligned} \quad \Rightarrow \quad \begin{aligned} x &= 2(-2T+3)-1 \\ y &= (-2T+3)^2 - 3(-2T+3) \end{aligned} \quad \Rightarrow \quad \begin{aligned} x &= -4T+5 \\ y &= 4T^2-6T \end{aligned} \text{ with } T \in [0, 2]$$

The final parametric equations and domain correspond to the curve on the right, as desired.